

1 Introduction

We have shown in several papers how the treatment of spin can be generalized[1-5]. For spin 1/2[1-3] and spin 1[5], we have derived generalized probability amplitudes for measurements on the systems, and we have obtained generalized forms of the components of the spin operators. In this paper, we extend this work to the case of spin 2. The method to be used in obtaining the generalized quantities is given in full detail in refs 1-5. Here the steps of the method will be used without description. The ideas underlying the method owe their genesis to the work of Landé[6-9].

2 Probability Amplitudes

To obtain the generalized probability amplitudes, we start with the standard expressions. These we derive by first obtaining the eigenvectors of the operator that results from taking the dot product of the standard operator and an arbitrary unit vector. Let the unit vector be $\hat{\mathbf{a}}$, with polar angles (θ, φ) . The standard operators for spin 2 are

$$[S_x] = \hbar \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & \frac{\sqrt{6}}{2} & 0 & 0 \\ 0 & \frac{\sqrt{6}}{2} & 0 & \frac{\sqrt{6}}{2} & 0 \\ 0 & 0 & \frac{\sqrt{6}}{2} & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad (1)$$

$$[S_y] = \hbar \begin{pmatrix} 0 & -i & 0 & 0 & 0 \\ i & 0 & -\frac{i\sqrt{6}}{2} & 0 & 0 \\ 0 & \frac{i\sqrt{6}}{2} & 0 & -\frac{i\sqrt{6}}{2} & 0 \\ 0 & 0 & \frac{i\sqrt{6}}{2} & 0 & -i \\ 0 & 0 & 0 & i & 0 \end{pmatrix} \quad (2)$$

and

$$[S_z] = \hbar \begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -2 \end{pmatrix} \quad (3)$$

so that $\mathbf{S} = \hat{\mathbf{i}}S_x + \hat{\mathbf{j}}S_y + \hat{\mathbf{k}}S_z$. Since $\hat{\mathbf{a}} = (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta)$, we have

$$[\hat{\mathbf{a}} \cdot \mathbf{S}] = \hbar \begin{pmatrix} 2 \cos \theta & \sin \theta e^{-i\varphi} & 0 & 0 & 0 \\ \sin \theta e^{i\varphi} & \cos \theta & \frac{\sqrt{6}}{2} \sin \theta e^{-i\varphi} & 0 & 0 \\ 0 & \frac{\sqrt{6}}{2} \sin \theta e^{i\varphi} & 0 & \frac{\sqrt{6}}{2} \sin \theta e^{-i\varphi} & 0 \\ 0 & 0 & \frac{\sqrt{6}}{2} \sin \theta e^{i\varphi} & -\cos \theta & \sin \theta e^{-i\varphi} \\ 0 & 0 & 0 & \sin \theta e^{i\varphi} & -2 \cos \theta \end{pmatrix} \quad (4)$$

The normalized eigenvectors of this matrix are

$$[\psi_2] = \begin{pmatrix} \cos^4 \frac{\theta}{2} e^{-i2\varphi} \\ \sin \theta \cos^2 \frac{\theta}{2} e^{-i\varphi} \\ \frac{\sqrt{6}}{4} \sin^2 \theta \\ \sin \theta \sin^2 \frac{\theta}{2} e^{i\varphi} \\ \sin^4 \frac{\theta}{2} e^{i2\varphi} \end{pmatrix} \quad (5)$$

$$[\psi_1] = \begin{pmatrix} \sin \theta \cos^2 \frac{\theta}{2} e^{-i2\varphi} \\ (3 \sin^2 \frac{\theta}{2} - \cos^2 \frac{\theta}{2}) \cos^2 \frac{\theta}{2} e^{-i\varphi} \\ -\frac{\sqrt{6}}{2} \sin \theta \cos \theta \\ -(3 \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}) \sin^2 \frac{\theta}{2} e^{i\varphi} \\ -\sin \theta \sin^2 \frac{\theta}{2} e^{i2\varphi} \end{pmatrix} \quad (6)$$

$$[\psi_0] = \begin{pmatrix} \frac{\sqrt{6}}{4} \sin^2 \theta e^{-i2\varphi} \\ -\frac{\sqrt{6}}{2} \sin \theta \cos \theta e^{-i\varphi} \\ \frac{1}{2} (2 \cos^2 \theta - \sin^2 \theta) \\ \frac{\sqrt{6}}{2} \sin \theta \cos \theta e^{i\varphi} \\ \frac{\sqrt{6}}{4} \sin^2 \theta e^{i2\varphi} \end{pmatrix} \quad (7)$$

$$[\psi_{-1}] = \begin{pmatrix} \sin \theta \sin^2 \frac{\theta}{2} e^{-i2\varphi} \\ -(3 \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}) \sin^2 \frac{\theta}{2} e^{-i\varphi} \\ \frac{\sqrt{6}}{2} \sin \theta \cos \theta \\ (3 \sin^2 \frac{\theta}{2} - \cos^2 \frac{\theta}{2}) \cos^2 \frac{\theta}{2} e^{i\varphi} \\ -\sin \theta \cos^2 \frac{\theta}{2} e^{i2\varphi} \end{pmatrix} \quad (8)$$

and

$$[\psi_{-2}] = \begin{pmatrix} \sin^4 \frac{\theta}{2} e^{-i2\varphi} \\ -\sin \theta \sin^2 \frac{\theta}{2} e^{-i\varphi} \\ \frac{\sqrt{6}}{4} \sin^2 \theta \\ -\sin \theta \cos^2 \frac{\theta}{2} e^{i\varphi} \\ \cos^4 \frac{\theta}{2} e^{i2\varphi} \end{pmatrix} \quad (9)$$

These vectors correspond to the case where the initial quantization direction is the vector $\hat{\mathbf{a}}$ (whose polar angles are (θ, φ)), while the final quantization direction is the z axis. As has been pointed out, the elements of this eigenvector are probability amplitudes of the form $\psi(m_i^{(\hat{\mathbf{a}})}; m_f^{(\hat{\mathbf{k}})})[1]$. The square modulus of the probability amplitude $\psi(m_i^{(\hat{\mathbf{a}})}; m_f^{(\hat{\mathbf{k}})})$ gives the probability that if the system is initially in a state of spin projection $m_i \hbar$ in the direction $\hat{\mathbf{a}}$, a measurement of the projection in the direction $\hat{\mathbf{k}}$ yields the value $m_f \hbar$. Hence, we are able to deduce that

$$\psi(2^{(\hat{\mathbf{a}})}; 2^{(\hat{\mathbf{k}})}) = \cos^4 \frac{\theta}{2} e^{-i2\varphi}, \quad (10)$$

$$\psi(2^{(\hat{\mathbf{a}})}; 1^{(\hat{\mathbf{k}})}) = \sin \theta \cos^2 \frac{\theta}{2} e^{-i\varphi}, \quad (11)$$

$$\psi(2^{(\hat{\mathbf{a}})}; 0^{(\hat{\mathbf{k}})}) = \frac{\sqrt{6}}{4} \sin^2 \theta, \quad (12)$$

$$\psi(2^{(\hat{\mathbf{a}})}; (-1)^{(\hat{\mathbf{k}})}) = \sin \theta \sin^2 \frac{\theta}{2} e^{i\varphi}, \quad (13)$$

$$\psi(2^{(\hat{\mathbf{a}})}; (-2)^{(\hat{\mathbf{k}})}) = \sin^4 \frac{\theta}{2} e^{i2\varphi}, \quad (14)$$

$$\psi(1^{(\hat{\mathbf{a}})}; 2^{(\hat{\mathbf{k}})}) = \sin \theta \cos^2 \frac{\theta}{2} e^{-i2\varphi}, \quad (15)$$

$$\psi(1^{(\hat{\mathbf{a}})}; 1^{(\hat{\mathbf{k}})}) = (3 \sin^2 \frac{\theta}{2} - \cos^2 \frac{\theta}{2}) \cos^2 \frac{\theta}{2} e^{-i\varphi}, \quad (16)$$

$$\psi(1^{(\hat{\mathbf{a}})}; 0^{(\hat{\mathbf{k}})}) = -\frac{\sqrt{6}}{2} \sin \theta \cos \theta, \quad (17)$$

$$\psi(1^{(\hat{\mathbf{a}})}; (-1)^{(\hat{\mathbf{k}})}) = -(3 \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}) \sin^2 \frac{\theta}{2} e^{i\varphi}, \quad (18)$$

$$\psi(1^{(\widehat{\mathbf{a}})}; (-2)^{(\widehat{\mathbf{k}})}) = -\sin \theta \sin^2 \frac{\theta}{2} e^{i2\varphi}, \quad (19)$$

$$\psi(0^{(\widehat{\mathbf{a}})}; 2^{(\widehat{\mathbf{k}})}) = \frac{\sqrt{6}}{4} \sin^2 \theta e^{-i2\varphi}, \quad (20)$$

$$\psi(0^{(\widehat{\mathbf{a}})}; 1^{(\widehat{\mathbf{k}})}) = -\frac{\sqrt{6}}{2} \sin \theta \cos \theta e^{-i\varphi}, \quad (21)$$

$$\psi(0^{(\widehat{\mathbf{a}})}; 0^{(\widehat{\mathbf{k}})}) = \frac{1}{2} (2 \cos^2 \theta - \sin^2 \theta), \quad (22)$$

$$\psi(0^{(\widehat{\mathbf{a}})}; (-1)^{(\widehat{\mathbf{k}})}) = \frac{\sqrt{6}}{2} \sin \theta \cos \theta e^{i\varphi}, \quad (23)$$

$$\psi(0^{(\widehat{\mathbf{a}})}; (-2)^{(\widehat{\mathbf{k}})}) = \frac{\sqrt{6}}{4} \sin^2 \theta e^{i2\varphi}, \quad (24)$$

$$\psi((-1)^{(\widehat{\mathbf{a}})}; 2^{(\widehat{\mathbf{k}})}) = \sin \theta \sin^2 \frac{\theta}{2} e^{-i2\varphi}, \quad (25)$$

$$\psi((-1)^{(\widehat{\mathbf{a}})}; 1^{(\widehat{\mathbf{k}})}) = -(3 \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}) \sin^2 \frac{\theta}{2} e^{-i\varphi}, \quad (26)$$

$$\psi((-1)^{(\widehat{\mathbf{a}})}; 0^{(\widehat{\mathbf{k}})}) = \frac{\sqrt{6}}{2} \sin \theta \cos \theta, \quad (27)$$

$$\psi((-1)^{(\widehat{\mathbf{a}})}; (-1)^{(\widehat{\mathbf{k}})}) = (3 \sin^2 \frac{\theta}{2} - \cos^2 \frac{\theta}{2}) \cos^2 \frac{\theta}{2} e^{i\varphi}, \quad (28)$$

$$\psi((-1)^{(\widehat{\mathbf{a}})}; (-2)^{(\widehat{\mathbf{k}})}) = -\sin \theta \cos^2 \frac{\theta}{2} e^{i2\varphi}, \quad (29)$$

$$\psi((-2)^{(\widehat{\mathbf{a}})}; 2^{(\widehat{\mathbf{k}})}) = \sin^4 \frac{\theta}{2} e^{-i2\varphi}, \quad (30)$$

$$\psi((-2)^{(\widehat{\mathbf{a}})}; 1^{(\widehat{\mathbf{k}})}) = -\sin \theta \sin^2 \frac{\theta}{2} e^{-i\varphi}, \quad (31)$$

$$\psi((-2)^{(\widehat{\mathbf{a}})}; 0^{(\widehat{\mathbf{k}})}) = \frac{\sqrt{6}}{4} \sin^2 \theta, \quad (32)$$

$$\psi((-2)^{(\widehat{\mathbf{a}})}; (-1)^{(\widehat{\mathbf{k}})}) = -\sin \theta \cos^2 \frac{\theta}{2} e^{i\varphi} \quad (33)$$

and

$$\psi((-2)^{(\widehat{\mathbf{a}})}; (-2)^{(\widehat{\mathbf{k}})}) = \cos^4 \frac{\theta}{2} e^{i2\varphi}. \quad (34)$$

3 Generalized Probability Amplitudes

We now consider the general case. Here, the original direction of quantization is given by the vector $\hat{\mathbf{a}}(\theta', \varphi')$ and the final direction by the vector $\hat{\mathbf{c}}(\theta, \varphi)$. Then the eigenvectors are given by

$$[\psi_{m_i}] = \begin{pmatrix} \psi(m_i^{\hat{\mathbf{a}}}; 2^{\hat{\mathbf{c}}}) \\ \psi(m_i^{\hat{\mathbf{a}}}; 1^{\hat{\mathbf{c}}}) \\ \psi(m_i^{\hat{\mathbf{a}}}; 0^{\hat{\mathbf{c}}}) \\ \psi(m_i^{\hat{\mathbf{a}}}; (-1)^{\hat{\mathbf{c}}}) \\ \psi(m_i^{\hat{\mathbf{a}}}; (-2)^{\hat{\mathbf{c}}}) \end{pmatrix} \quad (35)$$

To obtain the generalized probability amplitudes $\psi(m_i^{\hat{\mathbf{a}}}; m_f^{\hat{\mathbf{c}}})$, we use the expansion[6]

$$\psi(m_i^{\hat{\mathbf{a}}}; m_f^{\hat{\mathbf{c}}}) = \sum_{m=-2}^2 \psi(m_i^{\hat{\mathbf{a}}}; m^{\hat{\mathbf{b}}}) \psi(m^{\hat{\mathbf{b}}}; m_f^{\hat{\mathbf{c}}}) \quad (36)$$

where $\hat{\mathbf{b}}$ is an intermediate arbitrary direction, whose polar angles never appear in the final expressions. This is a particular form of the expansion

$$\psi(A_i, C_k) = \sum_{j=1}^N \chi(A_i, B_j) \phi(B_j, C_k). \quad (37)$$

which is always true for any three observables A , B and C of a system. Here ψ are the probability amplitudes for measurement of C from the state in which the system is initially in an eigenstate of A ; χ are the probability amplitudes for measurement of B for the same case. In the same way, ϕ are the probability amplitudes for measurement of C when the system is initially in an eigenstate of B . Here, A_i , B_i and C_i are eigenvalues of the observables A , B and C .

For the present case, the eigenvector for the eigenvalue $2\hbar$ is

$$[\psi_2] = \begin{pmatrix} \psi(2^{\hat{\mathbf{a}}}; 2^{\hat{\mathbf{c}}}) \\ \psi(2^{\hat{\mathbf{a}}}; 1^{\hat{\mathbf{c}}}) \\ \psi(2^{\hat{\mathbf{a}}}; 0^{\hat{\mathbf{c}}}) \\ \psi(2^{\hat{\mathbf{a}}}; (-1)^{\hat{\mathbf{c}}}) \\ \psi(2^{\hat{\mathbf{a}}}; (-2)^{\hat{\mathbf{c}}}) \end{pmatrix} \quad (38)$$

With the aid of Eqs. (10) - (34) and the expansion Eq. (36), we find that the generalized probability amplitudes are

$$\begin{aligned}
\psi(2^{(\widehat{\mathbf{a}})}; 2^{(\widehat{\mathbf{c}})}) &= \cos^4 \frac{\theta}{2} \cos^4 \frac{\theta'}{2} e^{i2(\varphi-\varphi')} \\
&+ \sin \theta' \sin \theta \cos^2 \frac{\theta'}{2} \cos^2 \frac{\theta}{2} e^{i(\varphi-\varphi')} + \frac{3}{8} \sin^2 \theta' \sin^2 \theta \\
&+ \sin \theta' \sin \theta \sin^2 \frac{\theta'}{2} \sin^2 \frac{\theta}{2} e^{-i(\varphi-\varphi')} \\
&+ \sin^4 \frac{\theta}{2} \sin^4 \frac{\theta'}{2} e^{-i2(\varphi-\varphi')}, \tag{39}
\end{aligned}$$

$$\begin{aligned}
\psi(2^{(\widehat{\mathbf{a}})}; 1^{(\widehat{\mathbf{c}})}) &= \sin \theta \cos^4 \frac{\theta'}{2} \cos^2 \frac{\theta}{2} e^{i2(\varphi-\varphi')} \\
&+ (3 \sin^2 \frac{\theta}{2} - \cos^2 \frac{\theta}{2}) \sin \theta' \cos^2 \frac{\theta'}{2} \cos^2 \frac{\theta}{2} e^{i(\varphi-\varphi')} \\
&- \frac{3}{4} \sin^2 \theta' \sin \theta \cos \theta \\
&- (3 \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}) \sin \theta' \sin^2 \frac{\theta'}{2} \sin^2 \frac{\theta}{2} e^{-i(\varphi-\varphi')} \\
&- \sin \theta \sin^4 \frac{\theta'}{2} \sin^2 \frac{\theta}{2} e^{-i2(\varphi-\varphi')} \tag{40}
\end{aligned}$$

$$\begin{aligned}
\psi(2^{(\widehat{\mathbf{a}})}; 0^{(\widehat{\mathbf{c}})}) &= \sqrt{\frac{3}{8}} \sin^2 \theta \cos^4 \frac{\theta'}{2} e^{i2(\varphi-\varphi')} \\
&- \sqrt{\frac{3}{2}} \sin \theta \sin \theta' \cos \theta \cos^2 \frac{\theta'}{2} e^{i(\varphi-\varphi')} + \sqrt{\frac{3}{32}} (2 \cos^2 \theta - \sin^2 \theta) \sin^2 \theta' \\
&+ \sqrt{\frac{3}{2}} \sin \theta \sin \theta' \cos \theta \sin^2 \frac{\theta'}{2} e^{-i(\varphi-\varphi')} + \sqrt{\frac{3}{8}} \sin^2 \theta \sin^4 \frac{\theta'}{2} e^{-i2(\varphi-\varphi')} \tag{41}
\end{aligned}$$

$$\begin{aligned}
\psi(2^{(\widehat{\mathbf{a}})}; (-1)^{(\widehat{\mathbf{c}})}) &= \sin \theta \cos^4 \frac{\theta'}{2} \sin^2 \frac{\theta}{2} e^{i2(\varphi-\varphi')} \\
&- (3 \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}) \sin \theta' \cos^2 \frac{\theta'}{2} \sin^2 \frac{\theta}{2} e^{i(\varphi-\varphi')}
\end{aligned}$$

$$\begin{aligned}
& +\frac{3}{4}\sin^2\theta'\sin\theta\cos\theta \\
& +(3\sin^2\frac{\theta}{2}-\cos^2\frac{\theta}{2})\sin\theta'\sin^2\frac{\theta'}{2}\cos^2\frac{\theta}{2}e^{-i(\varphi-\varphi')} \\
& -\sin\theta\sin^4\frac{\theta'}{2}\cos^2\frac{\theta}{2}e^{-i2(\varphi-\varphi')}
\end{aligned} \tag{42}$$

and

$$\begin{aligned}
\psi(2^{(\hat{\mathbf{a}})}; (-2)^{(\hat{\mathbf{c}})}) &= \sin^4\frac{\theta}{2}\cos^4\frac{\theta'}{2}e^{i2(\varphi-\varphi')} \\
& -\sin\theta'\sin\theta\cos^2\frac{\theta'}{2}\sin^2\frac{\theta}{2}e^{i(\varphi-\varphi')} + \frac{3}{8}\sin^2\theta'\sin^2\theta \\
& -\sin\theta'\sin\theta\sin^2\frac{\theta'}{2}\cos^2\frac{\theta}{2}e^{-i(\varphi-\varphi')} \\
& +\cos^4\frac{\theta}{2}\sin^4\frac{\theta'}{2}e^{-i2(\varphi-\varphi')}.
\end{aligned} \tag{43}$$

For the eigenvalue \hbar , the vector is

$$[\psi_1] = \begin{pmatrix} \psi(1^{(\hat{\mathbf{a}})}; 2^{(\hat{\mathbf{c}})}) \\ \psi(1^{(\hat{\mathbf{a}})}; 1^{(\hat{\mathbf{c}})}) \\ \psi(1^{(\hat{\mathbf{a}})}; 0^{(\hat{\mathbf{c}})}) \\ \psi(1^{(\hat{\mathbf{a}})}; (-1)^{(\hat{\mathbf{c}})}) \\ \psi(1^{(\hat{\mathbf{a}})}; (-2)^{(\hat{\mathbf{c}})}) \end{pmatrix} \tag{44}$$

where the probability amplitudes are

$$\begin{aligned}
\psi(1^{(\hat{\mathbf{a}})}; 2^{(\hat{\mathbf{c}})}) &= \sin\theta'\cos^2\frac{\theta'}{2}\cos^4\frac{\theta}{2}e^{i2(\varphi-\varphi')} \\
& +(3\sin^2\frac{\theta'}{2}-\cos^2\frac{\theta'}{2})\sin\theta\cos^2\frac{\theta'}{2}\cos^2\frac{\theta}{2}e^{i(\varphi-\varphi')} \\
& -\frac{3}{4}\sin^2\theta\sin\theta'\cos\theta' \\
& +(3\cos^2\frac{\theta'}{2}-\sin^2\frac{\theta'}{2})\sin\theta\sin^2\frac{\theta'}{2}\sin^2\frac{\theta}{2}e^{-i(\varphi-\varphi')} \\
& -\sin\theta'\sin^2\frac{\theta'}{2}\sin^4\frac{\theta}{2}e^{-i2(\varphi-\varphi')}
\end{aligned} \tag{45}$$

$$\psi(1^{(\hat{\mathbf{a}})}; 1^{(\hat{\mathbf{c}})}) = \sin\theta'\sin\theta\cos^2\frac{\theta'}{2}\cos^2\frac{\theta}{2}e^{i2(\varphi-\varphi')}$$

$$\begin{aligned}
& + (3 \sin^2 \frac{\theta'}{2} - \cos^2 \frac{\theta'}{2}) (3 \sin^2 \frac{\theta}{2} - \cos^2 \frac{\theta}{2}) \cos^2 \frac{\theta'}{2} \cos^2 \frac{\theta}{2} e^{i(\varphi - \varphi')} \\
& + \frac{3}{2} \sin \theta \sin \theta' \cos \theta \cos \theta' \\
& + (3 \cos^2 \frac{\theta'}{2} - \sin^2 \frac{\theta'}{2}) (3 \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}) \sin^2 \frac{\theta'}{2} \sin^2 \frac{\theta}{2} e^{-i(\varphi - \varphi')} \\
& + \sin \theta' \sin \theta \sin^2 \frac{\theta'}{2} \sin^2 \frac{\theta}{2} e^{-i2(\varphi - \varphi')}, \tag{46}
\end{aligned}$$

$$\begin{aligned}
\psi(1^{(\widehat{\mathbf{a}})}; 0^{(\widehat{\mathbf{c}})}) &= \sqrt{\frac{3}{8}} \sin \theta' \sin^2 \theta \cos^2 \frac{\theta'}{2} e^{i2(\varphi - \varphi')} \\
& - \sqrt{\frac{3}{2}} (3 \sin^2 \frac{\theta'}{2} - \cos^2 \frac{\theta'}{2}) \sin \theta \cos \theta \cos^2 \frac{\theta'}{2} e^{i(\varphi - \varphi')} \\
& - \sqrt{\frac{3}{8}} (2 \cos^2 \theta - \sin^2 \theta) \sin \theta' \cos \theta' \\
& - \sqrt{\frac{3}{2}} (3 \cos^2 \frac{\theta'}{2} - \sin^2 \frac{\theta'}{2}) \sin \theta \cos \theta \sin^2 \frac{\theta'}{2} e^{-i(\varphi - \varphi')} \\
& - \sqrt{\frac{3}{8}} \sin \theta' \sin^2 \theta \sin^2 \frac{\theta'}{2} e^{-i2(\varphi - \varphi')}, \tag{47}
\end{aligned}$$

and

$$\begin{aligned}
\psi(1^{(\widehat{\mathbf{a}})}; (-1)^{(\widehat{\mathbf{c}})}) &= \sin \theta' \sin \theta \cos^2 \frac{\theta'}{2} \sin^2 \frac{\theta}{2} e^{i2(\varphi - \varphi')} \\
& - (3 \sin^2 \frac{\theta'}{2} - \cos^2 \frac{\theta'}{2}) (3 \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}) \cos^2 \frac{\theta'}{2} \sin^2 \frac{\theta}{2} e^{i(\varphi - \varphi')} \\
& - \frac{3}{2} \sin \theta \sin \theta' \cos \theta \cos \theta' \\
& - (3 \cos^2 \frac{\theta'}{2} - \sin^2 \frac{\theta'}{2}) (3 \sin^2 \frac{\theta}{2} - \cos^2 \frac{\theta}{2}) \sin^2 \frac{\theta'}{2} \cos^2 \frac{\theta}{2} e^{-i(\varphi - \varphi')} \\
& + \sin \theta' \sin \theta \sin^2 \frac{\theta'}{2} \cos^2 \frac{\theta}{2} e^{-i2(\varphi - \varphi')}, \tag{48}
\end{aligned}$$

$$\psi(1^{(\widehat{\mathbf{a}})}; (-2)^{(\widehat{\mathbf{c}})}) = \sin \theta' \cos^2 \frac{\theta'}{2} \sin^4 \frac{\theta}{2} e^{i2(\varphi - \varphi')}$$

$$\begin{aligned}
& -(3 \sin^2 \frac{\theta'}{2} - \cos^2 \frac{\theta'}{2}) \sin \theta \cos^2 \frac{\theta'}{2} \sin^2 \frac{\theta}{2} e^{i(\varphi - \varphi')} \\
& - \frac{3}{4} \sin^2 \theta \sin \theta' \cos \theta' - \sin \theta' \sin^2 \frac{\theta'}{2} \cos^4 \frac{\theta}{2} e^{-i2(\varphi - \varphi')} \\
& + (3 \cos^2 \frac{\theta'}{2} - \sin^2 \frac{\theta'}{2}) \sin \theta \sin^2 \frac{\theta'}{2} \cos^2 \frac{\theta}{2} e^{-i(\varphi - \varphi')}, \tag{49}
\end{aligned}$$

For the eigenvalue 0, the vector is

$$[\psi_0] = \begin{pmatrix} \psi(0^{(\hat{\mathbf{a}})}; 2^{(\hat{\mathbf{c}})}) \\ \psi(0^{(\hat{\mathbf{a}})}; 1^{(\hat{\mathbf{c}})}) \\ \psi(0^{(\hat{\mathbf{a}})}; 0^{(\hat{\mathbf{c}})}) \\ \psi(0^{(\hat{\mathbf{a}})}; (-1)^{(\hat{\mathbf{c}})}) \\ \psi(0^{(\hat{\mathbf{a}})}; (-2)^{(\hat{\mathbf{c}})}) \end{pmatrix} \tag{50}$$

with the probability amplitudes

$$\begin{aligned}
\psi(0^{(\hat{\mathbf{a}})}; 2^{(\hat{\mathbf{c}})}) &= \sqrt{\frac{3}{8}} \sin^2 \theta' \cos^4 \frac{\theta}{2} e^{i2(\varphi - \varphi')} \\
& - \sqrt{\frac{3}{2}} \sin \theta \sin \theta' \cos \theta' \cos^2 \frac{\theta}{2} e^{i(\varphi - \varphi')} \\
& + \sqrt{\frac{3}{32}} (2 \cos^2 \theta' - \sin^2 \theta') \sin^2 \theta \\
& + \sqrt{\frac{3}{2}} \sin \theta \sin \theta' \cos \theta' \sin^2 \frac{\theta}{2} e^{-i(\varphi - \varphi')} \\
& + \sqrt{\frac{3}{8}} \sin^2 \theta' \sin^4 \frac{\theta}{2} e^{-i2(\varphi - \varphi')} \tag{51}
\end{aligned}$$

$$\begin{aligned}
\psi(0^{(\hat{\mathbf{a}})}; 1^{(\hat{\mathbf{c}})}) &= \sqrt{\frac{3}{8}} \sin \theta \sin^2 \theta' \cos^2 \frac{\theta}{2} e^{i2(\varphi - \varphi')} \\
& - \sqrt{\frac{3}{2}} (3 \sin^2 \frac{\theta}{2} - \cos^2 \frac{\theta}{2}) \sin \theta' \cos \theta' \cos^2 \frac{\theta}{2} e^{i(\varphi - \varphi')} \\
& - \sqrt{\frac{3}{8}} (2 \cos^2 \theta' - \sin^2 \theta') \sin \theta \cos \theta \\
& - \sqrt{\frac{3}{2}} (3 \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}) \sin \theta' \cos \theta' \sin^2 \frac{\theta}{2} e^{-i(\varphi - \varphi')}
\end{aligned}$$

$$-\sqrt{\frac{3}{8}} \sin \theta \sin^2 \theta' \sin^2 \frac{\theta}{2} e^{-i2(\varphi-\varphi')}, \quad (52)$$

$$\begin{aligned} \psi(0^{(\widehat{\mathbf{a}})}; 0^{(\widehat{\mathbf{c}})}) &= \frac{3}{8} \sin^2 \theta \sin^2 \theta' e^{i2(\varphi-\varphi')} \\ &+ \frac{3}{2} \sin \theta' \cos \theta' \sin \theta \cos \theta e^{i(\varphi-\varphi')} + \\ &+ \frac{3}{2} \sin \theta' \cos \theta' \sin \theta \cos \theta e^{-i(\varphi-\varphi')} \\ &+ \frac{1}{4} (2 \cos^2 \theta' - \sin^2 \theta') (2 \cos^2 \theta - \sin^2 \theta) \\ &+ \frac{3}{8} \sin^2 \theta \sin^2 \theta' e^{-i2(\varphi-\varphi')}, \end{aligned} \quad (53)$$

and

$$\begin{aligned} \psi(0^{(\widehat{\mathbf{a}})}; (-1)^{(\widehat{\mathbf{c}})}) &= \sqrt{\frac{3}{8}} \sin \theta \sin^2 \theta' \sin^2 \frac{\theta}{2} e^{i2(\varphi-\varphi')} \\ &+ \sqrt{\frac{3}{2}} (3 \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}) \sin \theta' \cos \theta' \sin^2 \frac{\theta}{2} e^{i(\varphi-\varphi')} \\ &+ \sqrt{\frac{3}{2}} (3 \sin^2 \frac{\theta}{2} - \cos^2 \frac{\theta}{2}) \sin \theta' \cos \theta' \cos^2 \frac{\theta}{2} e^{-i(\varphi-\varphi')} \\ &+ \sqrt{\frac{3}{8}} (2 \cos^2 \theta' - \sin^2 \theta') \sin \theta \cos \theta \\ &- \sqrt{\frac{3}{8}} \sin \theta \sin^2 \theta' \cos^2 \frac{\theta}{2} e^{-i2(\varphi-\varphi')}, \end{aligned} \quad (54)$$

$$\begin{aligned} \psi(0^{(\widehat{\mathbf{a}})}; (-2)^{(\widehat{\mathbf{c}})}) &= \sqrt{\frac{3}{8}} \sin^2 \theta' \sin^4 \frac{\theta}{2} e^{i2(\varphi-\varphi')} \\ &+ \sqrt{\frac{3}{2}} \sin \theta \sin \theta' \cos \theta' \sin^2 \frac{\theta}{2} e^{i(\varphi-\varphi')} + \sqrt{\frac{3}{8}} \sin^2 \theta' \cos^4 \frac{\theta}{2} e^{-i2(\varphi-\varphi')} \\ &+ \sqrt{\frac{3}{32}} (2 \cos^2 \theta' - \sin^2 \theta') \sin^2 \theta \\ &+ \sqrt{\frac{3}{2}} \sin \theta \sin \theta' \cos \theta' \cos^2 \frac{\theta}{2} e^{-i(\varphi-\varphi')}, \end{aligned} \quad (55)$$

Finally, for the eigenvalue $-\hbar$, the vector is

$$[\psi_{-1}] = \begin{pmatrix} \psi((-1)^{\widehat{\mathbf{a}}}; 2^{\widehat{\mathbf{c}}}) \\ \psi((-1)^{\widehat{\mathbf{a}}}; 1^{\widehat{\mathbf{c}}}) \\ \psi((-1)^{\widehat{\mathbf{a}}}; 0^{\widehat{\mathbf{c}}}) \\ \psi((-1)^{\widehat{\mathbf{a}}}; (-1)^{\widehat{\mathbf{c}}}) \\ \psi((-1)^{\widehat{\mathbf{a}}}; (-2)^{\widehat{\mathbf{c}}}) \end{pmatrix} \quad (56)$$

where the probability amplitudes are

$$\begin{aligned} \psi((-1)^{\widehat{\mathbf{a}}}; 2^{\widehat{\mathbf{c}}}) &= -\sin \theta' \sin^2 \frac{\theta'}{2} \cos^4 \frac{\theta}{2} e^{i2(\varphi-\varphi')} \\ &\quad - (3 \cos^2 \frac{\theta'}{2} - \sin^2 \frac{\theta'}{2}) \sin \theta \sin^2 \frac{\theta'}{2} \cos^2 \frac{\theta}{2} e^{i(\varphi-\varphi')} \\ &\quad + \frac{3}{4} \sin^2 \theta \sin \theta' \cos \theta' - \sin \theta' \cos^2 \frac{\theta'}{2} \sin^4 \frac{\theta}{2} e^{-i2(\varphi-\varphi')} \\ &\quad + (3 \sin^2 \frac{\theta'}{2} - \cos^2 \frac{\theta'}{2}) \sin \theta \cos^2 \frac{\theta'}{2} \sin^2 \frac{\theta}{2} e^{-i(\varphi-\varphi')}, \end{aligned} \quad (57)$$

$$\begin{aligned} \psi((-1)^{\widehat{\mathbf{a}}}; 1^{\widehat{\mathbf{c}}}) &= \sin \theta' \sin \theta \sin^2 \frac{\theta'}{2} \cos^2 \frac{\theta}{2} e^{i2(\varphi-\varphi')} \\ &\quad - (3 \cos^2 \frac{\theta'}{2} - \sin^2 \frac{\theta'}{2}) (3 \sin^2 \frac{\theta}{2} - \cos^2 \frac{\theta}{2}) \sin^2 \frac{\theta'}{2} \cos^2 \frac{\theta}{2} e^{i(\varphi-\varphi')} \\ &\quad - (3 \sin^2 \frac{\theta'}{2} - \cos^2 \frac{\theta'}{2}) (3 \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}) \cos^2 \frac{\theta'}{2} \sin^2 \frac{\theta}{2} e^{-i(\varphi-\varphi')} \\ &\quad - \frac{3}{2} \sin \theta \sin \theta' \cos \theta \cos \theta' + \sin \theta' \sin \theta \cos^2 \frac{\theta'}{2} \sin^2 \frac{\theta}{2} e^{-i2(\varphi-\varphi')}, \end{aligned} \quad (58)$$

$$\begin{aligned} \psi((-1)^{\widehat{\mathbf{a}}}; 0^{\widehat{\mathbf{c}}}) &= \sqrt{\frac{3}{8}} \sin \theta' \sin^2 \theta \sin^2 \frac{\theta'}{2} e^{i2(\varphi-\varphi')} \\ &\quad + \sqrt{\frac{3}{2}} (3 \cos^2 \frac{\theta'}{2} - \sin^2 \frac{\theta'}{2}) \sin \theta \cos \theta \sin^2 \frac{\theta'}{2} e^{i(\varphi-\varphi')} \\ &\quad + \sqrt{\frac{3}{2}} (3 \sin^2 \frac{\theta'}{2} - \cos^2 \frac{\theta'}{2}) \sin \theta \cos \theta \cos^2 \frac{\theta'}{2} e^{-i(\varphi-\varphi')} \end{aligned}$$

$$\begin{aligned}
& +\sqrt{\frac{3}{8}}(2\cos^2\theta - \sin^2\theta)\sin\theta'\cos\theta' \\
& +\sqrt{\frac{3}{8}}\sin\theta'\sin^2\theta\cos^2\frac{\theta'}{2}e^{-i2(\varphi-\varphi')}, \tag{59}
\end{aligned}$$

$$\begin{aligned}
\psi((-1)^{\widehat{\mathbf{a}}}; (-1)^{\widehat{\mathbf{c}}}) &= \sin\theta'\sin\theta\sin^2\frac{\theta'}{2}\sin^2\frac{\theta}{2}e^{i2(\varphi-\varphi')} \\
& + (3\cos^2\frac{\theta'}{2} - \sin^2\frac{\theta'}{2})(3\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2})\sin^2\frac{\theta'}{2}\sin^2\frac{\theta}{2}e^{i(\varphi-\varphi')} \\
& + (3\sin^2\frac{\theta'}{2} - \cos^2\frac{\theta'}{2})(3\sin^2\frac{\theta}{2} - \cos^2\frac{\theta}{2})\cos^2\frac{\theta'}{2}\cos^2\frac{\theta}{2}e^{-i(\varphi-\varphi')} \\
& + \frac{3}{2}\sin\theta\sin\theta'\cos\theta\cos\theta' + \sin\theta'\sin\theta\cos^2\frac{\theta'}{2}\cos^2\frac{\theta}{2}e^{-i2(\varphi-\varphi')} \tag{60}
\end{aligned}$$

and

$$\begin{aligned}
\psi((-1)^{\widehat{\mathbf{a}}}; (-2)^{\widehat{\mathbf{c}}}) &= \sin\theta'\sin^2\frac{\theta'}{2}\sin^4\frac{\theta}{2}e^{i2(\varphi-\varphi')} \\
& + (3\cos^2\frac{\theta'}{2} - \sin^2\frac{\theta'}{2})\sin\theta\sin^2\frac{\theta'}{2}\sin^2\frac{\theta}{2}e^{i(\varphi-\varphi')} \\
& + \frac{3}{4}\sin^2\theta\sin\theta'\cos\theta' - \sin\theta'\cos^2\frac{\theta'}{2}\cos^4\frac{\theta}{2}e^{-i2(\varphi-\varphi')} \\
& - (3\sin^2\frac{\theta'}{2} - \cos^2\frac{\theta'}{2})\sin\theta\cos^2\frac{\theta'}{2}\cos^2\frac{\theta}{2}e^{-i(\varphi-\varphi')}, \tag{61}
\end{aligned}$$

For the eigenvalue $-2\hbar$, the eigenvector is

$$[\psi_{-2}] = \begin{pmatrix} \psi((-2)^{\widehat{\mathbf{a}}}; 2^{\widehat{\mathbf{c}}}) \\ \psi((-2)^{\widehat{\mathbf{a}}}; 1^{\widehat{\mathbf{c}}}) \\ \psi((-2)^{\widehat{\mathbf{a}}}; 0^{\widehat{\mathbf{c}}}) \\ \psi((-2)^{\widehat{\mathbf{a}}}; (-1)^{\widehat{\mathbf{c}}}) \\ \psi((-2)^{\widehat{\mathbf{a}}}; (-2)^{\widehat{\mathbf{c}}}) \end{pmatrix} \tag{62}$$

so that the probability amplitudes are

$$\begin{aligned}
\psi((-2)^{\widehat{\mathbf{a}}}; 2^{\widehat{\mathbf{c}}}) &= \cos^4\frac{\theta}{2}\sin^4\frac{\theta'}{2}e^{i2(\varphi-\varphi')} \\
& - \sin\theta'\sin\theta\sin^2\frac{\theta'}{2}\cos^2\frac{\theta}{2}e^{i(\varphi-\varphi')} + \frac{3}{8}\sin^2\theta'\sin^2\theta
\end{aligned}$$

$$\begin{aligned}
& -\sin \theta' \sin \theta \cos^2 \frac{\theta'}{2} \sin^2 \frac{\theta}{2} e^{-i(\varphi-\varphi')} \\
& + \sin^4 \frac{\theta}{2} \cos^4 \frac{\theta'}{2} e^{-i2(\varphi-\varphi')}, \tag{63}
\end{aligned}$$

$$\begin{aligned}
\psi((-2)^{(\widehat{\mathbf{a}})}; 1^{(\widehat{\mathbf{c}})}) &= \sin \theta \sin^4 \frac{\theta'}{2} \cos^2 \frac{\theta}{2} e^{i2(\varphi-\varphi')} \\
& - (3 \sin^2 \frac{\theta}{2} - \cos^2 \frac{\theta}{2}) \sin \theta' \sin^2 \frac{\theta'}{2} \cos^2 \frac{\theta}{2} e^{i(\varphi-\varphi')} \\
& - \frac{3}{4} \sin^2 \theta' \sin \theta \cos \theta - \sin \theta \cos^4 \frac{\theta'}{2} \sin^2 \frac{\theta}{2} e^{-i2(\varphi-\varphi')} \\
& + (3 \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}) \sin \theta' \cos^2 \frac{\theta'}{2} \sin^2 \frac{\theta}{2} e^{-i(\varphi-\varphi')}, \tag{64}
\end{aligned}$$

$$\begin{aligned}
\psi((-2)^{(\widehat{\mathbf{a}})}; 0^{(\widehat{\mathbf{c}})}) &= \sqrt{\frac{3}{8}} \sin^2 \theta \sin^4 \frac{\theta'}{2} e^{i2(\varphi-\varphi')} \\
& + \sqrt{\frac{3}{2}} \sin \theta \sin \theta' \cos \theta \sin^2 \frac{\theta'}{2} e^{i(\varphi-\varphi')} + \sqrt{\frac{3}{8}} \sin^2 \theta \cos^4 \frac{\theta'}{2} e^{-i2(\varphi-\varphi')} \\
& + \sqrt{\frac{3}{32}} (2 \cos^2 \theta - \sin^2 \theta) \sin^2 \theta' \\
& - \sqrt{\frac{3}{2}} \sin \theta \sin \theta' \cos \theta \cos^2 \frac{\theta'}{2} e^{-i(\varphi-\varphi')}, \tag{65}
\end{aligned}$$

$$\begin{aligned}
\psi((-2)^{(\widehat{\mathbf{a}})}; (-1)^{(\widehat{\mathbf{c}})}) &= \sin \theta \sin^4 \frac{\theta'}{2} \sin^2 \frac{\theta}{2} e^{i2(\varphi-\varphi')} \\
& + (3 \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}) \sin \theta' \sin^2 \frac{\theta'}{2} \sin^2 \frac{\theta}{2} e^{i(\varphi-\varphi')} \\
& + \frac{3}{4} \sin^2 \theta' \sin \theta \cos \theta - \sin \theta \cos^4 \frac{\theta'}{2} \cos^2 \frac{\theta}{2} e^{-i2(\varphi-\varphi')} \\
& - (3 \sin^2 \frac{\theta}{2} - \cos^2 \frac{\theta}{2}) \sin \theta' \cos^2 \frac{\theta'}{2} \cos^2 \frac{\theta}{2} e^{-i(\varphi-\varphi')}, \tag{66}
\end{aligned}$$

and

$$\psi((-2)^{(\widehat{\mathbf{a}})}; (-2)^{(\widehat{\mathbf{c}})}) = \cos^4 \frac{\theta}{2} \cos^4 \frac{\theta'}{2} e^{i2(\varphi-\varphi')}$$

$$\begin{aligned}
& + \sin \theta' \sin \theta \sin^2 \frac{\theta'}{2} \sin^2 \frac{\theta}{2} e^{i(\varphi-\varphi')} + \frac{3}{8} \sin^2 \theta' \sin^2 \theta \\
& + \sin \theta' \sin \theta \cos^2 \frac{\theta'}{2} \cos^2 \frac{\theta}{2} e^{-i(\varphi-\varphi')} \\
& + \cos^4 \frac{\theta}{2} \cos^4 \frac{\theta'}{2} e^{-i2(\varphi-\varphi')}.
\end{aligned} \tag{67}$$

4 Discussion and Conclusion

In this paper, we have presented the generalized probability amplitudes for the case spin 2. The associated probabilities are obtained straightforwardly, if tediously, as the square moduli of these probability amplitudes. As we have emphasized, the approach to quantum mechanics that allows us to derive and interpret these quantities leads to a deeper understanding of quantum theory and will no doubt find application in future. One application of the probability amplitudes for spin 2 is in the derivation of generalized spherical harmonics; we have done this already for $l = 1$ [10]. The case $l = 2$ has already been treated and will shortly be the subject of another article. Another is in the treatment of angular momentum addition. We have shown in previous papers how to generalize spin and/or angular momentum addition[11,12], and this requires the generalized probability amplitudes for the spin system being treated.

As demonstrated in our work on spin 1/2 and spin 1, generalized probability amplitudes imply generalized forms of the spin operators. The form Eq. (4), which is considered in the literature to be the most generalized form of the operator for the z component is not so in reality. The true generalized form will be presented in a future paper.

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